

# Road Vehicle Dynamics

## Table of Contents:

### Foreword

### Preface

### Chapter 1 Introduction

- 1.1 General
- 1.2 Vehicle System Classification
- 1.3 Dynamic System
- 1.4 Classification of Dynamic System Models
- 1.5 Constraints, Generalized Coordinates, and Degrees of Freedom
- 1.6 Discrete and Continuous Systems
- 1.7 Vibration Analysis
- 1.8 Elements of Vibrating Systems
  - 1.8.1 Spring Elements
  - 1.8.2 Potential Energy of Linear Springs
  - 1.8.3 Equivalent Springs
    - 1.8.3.1 Springs in Parallel
    - 1.8.3.2 Springs in Series
  - 1.8.4 Mass or Inertia Elements
  - 1.8.5 Damping Elements
    - 1.8.5.1 Viscous Damping
    - 1.8.5.2 Coulomb Damping
    - 1.8.5.3 Structural or Hysteretic Damping
    - 1.8.5.4 Combination of Damping Elements
- 1.9 Review of Dynamics
  - 1.9.1 Newton's Laws of Motion
  - 1.9.2 Kinematics of Rigid Bodies
  - 1.9.3 Linear Momentum
  - 1.9.4 Principle of Conservation of Linear Momentum
  - 1.9.5 Angular Momentum
  - 1.9.6 Equations of Motion for a Rigid Body
  - 1.9.7 Angular Momentum of a Rigid Body
  - 1.9.8 Principle of Work and Energy
  - 1.9.9 Conservation of Energy
  - 1.9.10 Principle of Impulse and Momentum
  - 1.9.11 Mechanical Systems
  - 1.9.12 Translational Systems
  - 1.9.13 Rotational Systems

- 1.9.14 Translation and Rotational Systems
- 1.9.15 Angular Momentum and Moments of Inertia
- 1.9.16 Geared Systems
- 1.10 Lagrange's Equation
  - 1.10.1 Degrees of Freedom
  - 1.10.2 Generalized Coordinates
  - 1.10.3 Constraints
  - 1.10.4 Principle of Virtual Work
  - 1.10.5 D' Alembert's Principle
  - 1.10.6 Generalized Force
  - 1.10.7 Lagrange's Equations of Motion
  - 1.10.8 Holonomic Systems
  - 1.10.9 Nonholonomic Systems
  - 1.10.10 Rayleigh's Dissipation Function
- 1.11 Summary
- 1.12 References

### Chapter 2 Analysis of Dynamic Systems

- 2.1 Introduction
- 2.2 Classification of Vibrations
- 2.3 Classification of Deterministic Data
  - 2.3.1 Sinusoidal Periodic Data
  - 2.3.2 Complex Periodic Data
  - 2.3.3 Almost Periodic Data
  - 2.3.4 Transient Nonperiodic Data
- 2.4 Linear Dynamic Systems
  - 2.4.1 Linear Single-Degree-of-Freedom System
  - 2.4.2 Free Vibration of a Single-Degree-of-Freedom System
  - 2.4.3 Forced Vibration of a Single-Degree-of-Freedom System
  - 2.4.4 Linear Multiple-Degrees-of-Freedom System
  - 2.4.5 Eigenvalues and Eigenvectors: Undamped System
  - 2.4.6 Eigenvalues and Eigenvectors: Damped System
  - 2.4.7 Forced Vibration Solution of a Multiple-Degrees-of-Freedom System

# Road Vehicle Dynamics

- 2.5 Nonlinear Dynamic Systems
  - 2.5.1 Exact Methods for Nonlinear Systems
  - 2.5.2 Approximate Methods for Nonlinear Systems
    - 2.5.2.1 Iterative Method
    - 2.5.2.2 Ritz Averaging Method
    - 2.5.2.3 Perturbation Method
    - 2.5.2.4 Variation of Parameter Method
  - 2.5.3 Graphical Method
    - 2.5.3.1 Phase Plane Representation
    - 2.5.3.2 Phase Velocity
    - 2.5.3.3 Pell's Method
  - 2.5.4 Multiple-Degrees-of-Freedom Systems
- 2.6 Random Vibrations
  - 2.6.1 Probability Density Function
  - 2.6.2 Autocorrelation Function
- 2.7 Gaussian Random Process
  - 2.7.1 Fourier Analysis
    - 2.7.1.1 Fourier Series
    - 2.7.1.2 Fourier Integral
  - 2.7.2 Response of a Single-Degree-of-Freedom Vibrating System
    - 2.7.2.1 Impulse Response Method
    - 2.7.2.2 Frequency Response Method
  - 2.7.3 Power Spectral Density Function
  - 2.7.4 Joint Probability Density Function
  - 2.7.5 Cross-Correlation Function
  - 2.7.6 Application of Power Spectral Densities to Vehicle Dynamics
  - 2.7.7 Response of a Single-Degree-of-Freedom System to Random Inputs
  - 2.7.8 Response of Multiple- Degrees- of-Freedom Systems to Random Inputs
- 2.8 Summary
- 2.9 References

## Chapter 3 Tire Dynamics

- 3.1 Introduction
- 3.2 Vertical Dynamics of Tires
  - 3.2.1 Vertical Stiffness and Damping Characteristics of Tires
  - 3.2.2 Vertical Vibration Mechanics Models of Tires
    - 3.2.2.1 Point Contact Model of Tires
    - 3.2.2.2 Fixed Contact Patch Model of Tires
    - 3.2.2.3 Time-Varying Contact Patch Model of Tires
  - 3.2.3 Enveloping Characteristics of Tires
- 3.3 Tire Longitudinal Dynamics
  - 3.3.1 Tire Rolling Resistance

- 3.3.2 Rolling Resistance of the Tire with Toe-In
- 3.3.3 Rolling Resistance of the Turning Wheel
- 3.3.4 Longitudinal Adhesion Coefficient
- 3.3.5 Theoretical Model of Tire Longitudinal Force Under Driving and Braking
- 3.4 Tire Lateral Dynamics
  - 3.4.1 Tire Cornering Characteristics
  - 3.4.2 Mathematical Model of the Tire Cornering Characteristic
    - 3.4.2.1 Simplified Mathematical Model of the Tire Cornering Characteristic
    - 3.4.2.2 Cornering Characteristic with Lateral Bending Deformation of the Tire Case
  - 3.4.3 Rolling Properties of Tires
    - 3.4.3.1 Cambered Tire Models
    - 3.4.3.2 Cambered Tire Model with Roll Elastic Deformation of the Tire Carcass
- 3.5 Tire Mechanics Model Considering Longitudinal Slip and Cornering Characteristics
  - 3.5.1 C.G. Gim Theoretical Model
  - 3.5.2 K.H. Guo Tire Model
    - 3.5.2.1 Steady-State Simplified Theoretical Tire Model
    - 3.5.2.2 Nonsteady-State Semi-Empirical Tire Mechanics Model
  - 3.5.3 H.B. Pacejka Magic Formula Model
- 3.6 References

## Chapter 4 Ride Dynamics

- 4.1 Introduction
- 4.2 Vibration Environment in Road Vehicles
  - 4.2.1 Vibration Sources from the Road
    - 4.2.1.1 Power Spectral Density in Spatial Frequency
    - 4.2.1.2 Power Spectral Density in Temporal Frequency
  - 4.2.2 Vehicle Internal Vibration Sources
    - 4.2.2.1 Vibration Sources from the Powerplant
      - 4.2.2.1.1 Coordinates and Powerplant Modes
      - 4.2.2.1.2 Vibration Sources from Engine Firing Pulsation
      - 4.2.2.1.3 Vibration Sources from Powerplant Inertia Forces and Moments
      - 4.2.2.1.4 Powerplant Isolation Design
    - 4.2.2.2 Vibration Sources from the Driveline

# Road Vehicle Dynamics

- 4.2.2.2.1 Driveline Imbalance
- 4.2.2.2.2 Gear Transmission Error
- 4.2.2.2.3 Second Order Excitation
- 4.2.2.2.4 Driveshaft Modes and Driveline Modes
- 4.2.2.3 Vibration Sources from the Exhaust System
- 4.3 Vehicle Ride Models
  - 4.3.1 Quarter Car Model
    - 4.3.1.1 Modeling for the Quarter Car Model
    - 4.3.1.2 Modal Analysis for the Quarter Car Model
    - 4.3.1.3 Dynamic Analysis for the Quarter Car Model
      - 4.3.1.3.1 Transmissibility Between the Body Response and Road Excitation
      - 4.3.1.3.2 Transmissibility Between the Body Response and Vehicle Excitation
      - 4.3.1.3.3 Dynamic Response at Random Input
  - 4.3.2 Bounce-Pitch Model
  - 4.3.3 Other Modeling
- 4.4 Seat Evaluation and Modeling
  - 4.4.1 Introduction
  - 4.4.2 SEAT Value
  - 4.4.3 Seat Velocity
  - 4.4.4 Linear Seat Modeling and Transmissibility
  - 4.4.5 Nonlinear Seat Modeling and Transmissibility
- 4.5 Discomfort Evaluation and Human Body Model
  - 4.5.1 Discomfort and Subjective Evaluation
  - 4.5.2 Objective Evaluation of Ride Discomfort
    - 4.5.2.1 Weighted Root-Mean-Square Method
    - 4.5.2.2 Objective Evaluation by the Vibration Dose Value
  - 4.5.3 Linear Human Body Modeling
  - 4.5.4 Objective Evaluation by Nonlinear Seat-Human Body Modeling
- 4.6 Active and Semi-Active Control
  - 4.6.1 Introduction
  - 4.6.2 Basic Control Concepts
  - 4.6.3 Active Control
  - 4.6.4 Semi-Active Control
- 4.7 Summary
- 4.8 References

## Chapter 5 Vehicle Rollover Analysis

- 5.1 Introduction
  - 5.1.1 Rollover Scenario
  - 5.1.2 Importance of Rollover
  - 5.1.3 Research on Rollover
  - 5.1.4 Scope of This Chapter
- 5.2 Rigid Vehicle Rollover Model
  - 5.2.1 Rigid Vehicle Model
  - 5.2.2 Steady-State Rollover on a Flat Road
  - 5.2.3 Tilt Table Ratio
  - 5.2.4 Side Pull Ratio
- 5.3 Suspended Vehicle Rollover Model
  - 5.3.1 Steady-State Rollover Model for a Suspended Vehicle
  - 5.3.2 Contribution from the Tire Deflection
  - 5.3.3 Contribution from the Suspension Deflection
  - 5.3.4 Parameters Influencing the Suspended Rollover Model
- 5.4 Dynamic Rollover Model
  - 5.4.1 Rigid Dynamic Model
  - 5.4.2 Dynamic Rollover Model for a Dependent Suspension Vehicle
  - 5.4.3 Dynamic Rollover Model for an Independent Suspension Vehicle
  - 5.4.4 Rollover Simulation Tools
- 5.5 Dynamic Rollover Threshold
  - 5.5.1 Dynamic Stability Index
  - 5.5.2 Rollover Prevention Energy Reserve
  - 5.5.3 Rollover Prevention Metric
  - 5.5.4 Critical Sliding Velocity
- 5.6 Occupant in Rollover
  - 5.6.1 Overview of the Occupant and Rollover
  - 5.6.2 Testing of an Occupant Model
  - 5.6.3 Simulation of Occupant Rollover
- 5.7 Safety and Rollover Control
  - 5.7.1 Overview of Rollover Safety
  - 5.7.2 Sensing of Rollover
  - 5.7.3 Rollover Safety Control
- 5.8 Summary
- 5.9 References

## Chapter 6 Handling Dynamics

- 6.1 Introduction
  - 6.1.1 Tire Cornering Forces
  - 6.1.2 Forces and Torques in the Tire Contact Area
- 6.2 The Simplest Handling Models-Two-Degrees-of-Freedom Yaw Plane Model
- 6.3 Steady-State Handling Characteristics

# Road Vehicle Dynamics

- 6.3.1 Yaw Velocity Gain and Understeer Gradient
    - 6.3.1.1 Neutral Steer
    - 6.3.1.2 Understeer
    - 6.3.1.3 Oversteer
  - 6.3.2 Difference Between Slip Angles of the Front and Rear Wheels
  - 6.3.3 Ratio of Radius of Turn
  - 6.4 Dynamic Characteristics of Handling
    - 6.4.1 Handling Damping and Natural Frequency
    - 6.4.2 Step Steer Input Response
    - 6.4.3 Ramp Steer Input Response
    - 6.4.4 Impulse Input Excitation Response
    - 6.4.5 Frequency Response of Yaw Velocity
    - 6.4.6 Stability Analysis
    - 6.4.7 Curvature Response
  - 6.5 Chassis System Effects on Handling Characteristics
    - 6.5.1 Lateral Force Transfer Effects on Cornering
    - 6.5.2 Steering System
    - 6.5.3 Camber Change Effect
    - 6.5.4 Roll Steer Effect
    - 6.5.5 Lateral Force Compliance Steer
    - 6.5.6 Aligning Torque Effects
    - 6.5.7 Effect of Tractive Forces on Cornering
  - 6.6 Handling Safety-Overturning Limit Handling Characteristics
  - 6.7 Nonlinear Models of Handling Dynamics
    - 6.7.1 Multiple-Degrees-of-Freedom System Models
    - 6.7.2 An Eight-Degrees-of-Freedom System Model
  - 6.8 Testing of Handling Characteristics
    - 6.8.1 Constant Radius Turn
    - 6.8.2 Constant Speed Test
    - 6.8.3 Constant Steer Angle Test
      - 6.8.3.1 Dynamic Testing
      - 6.8.3.2 Simulations and Testing Validation
  - 6.9 Summary
  - 6.10 References
- Chapter 7 Braking**
- 7.1 Introduction
    - 7.1.1 Types of Automotive Brakes
    - 7.1.2 Braking Distance and Deceleration
  - 7.2 Brake Torque Distribution
    - 7.2.1 Drum Brakes
      - 7.2.1.1 Mechanical Advantage
      - 7.2.1.2 Torque Calculations
    - 7.2.2 Disk Brakes
    - 7.2.3 Consideration of Temperature
  - 7.3 Load Transfer During Braking
    - 7.3.1 Simple Braking on a Horizontal Road
    - 7.3.2 Effect of Aerodynamic and Other Forces
      - 7.3.2.1 Rolling Resistance
      - 7.3.2.2 Aerodynamic Drag
      - 7.3.2.3 Powertrain Resistance
      - 7.3.2.4 Load Transfer on a Horizontal Plane
    - 7.3.3 Effect of Grade
  - 7.4 Optimal Braking Performance
    - 7.4.1 Braking of a Single Axle
      - 7.4.1.1 Braking of the Front Axle
      - 7.4.1.2 Braking of the Rear Axle
      - 7.4.1.3 Safety Considerations
    - 7.4.2 Braking at Both Axles
      - 7.4.2.1 Front Lock-Up
      - 7.4.2.2 Rear Lock-Up
    - 7.4.3 Achieving Optimal Braking Performance
  - 7.5 Considerations of Vehicle Safety
    - 7.5.1 Skid (Slip) Condition and Braking
    - 7.5.2 Anti-Lock Braking System
  - 7.6 Pitch Plane Models
  - 7.7 Recent Advances in Automotive Braking
  - 7.8 Summary
  - 7.9 References
- Chapter 8 Acceleration**
- 8.1 Introduction
  - 8.2 Load Transfer During Acceleration
    - 8.2.1 Simple Acceleration on a Horizontal Road
    - 8.2.2 Effect of Aerodynamic and Other Forces
    - 8.2.3 Effect of Grade
  - 8.3 Traction-Limited Acceleration
    - 8.3.1 Drivetrain Configurations
    - 8.3.2 Front-Wheel Drive
    - 8.3.3 Rear-Wheel Drive
    - 8.3.4 All- Wheel-Drive and Four-by-Four Systems
      - 8.3.4.1 Front Skid
      - 8.3.4.2 Rear Skid
    - 8.3.5 Optimal Tractive Effort
  - 8.4 Power-Limited Acceleration
    - 8.4.1 The Engine
    - 8.4.2 Internal Combustion Engines
    - 8.4.3 The Transmission
      - 8.4.3.1 Manual Transmissions
      - 8.4.3.2 Automatic Transmissions

# Road Vehicle Dynamics

- 8.4.3.3 Continuously Variable Transmissions
- 8.4.4 Vehicle Acceleration
- 8.5 Safety Features
  - 8.5.1 Limited Slip Axle
  - 8.5.2 Traction Control
- 8.6 Summary
- 8.7 References

## Chapter 9 Total Vehicle Dynamics

- 9.1 Introduction
  - 9.1.1 Subjective and Objective Evaluations
  - 9.1.2 Target Setting
  - 9.1.3 Vehicle Dynamics Tests and Evaluations
    - 9.1.3.1 Ride
    - 9.1.3.2 Steering
    - 9.1.3.3 Handling
    - 9.1.3.4 Braking
    - 9.1.3.5 Performance
- 9.2 Steering and Braking
  - 9.2.1 Simple Braking and Steering on a Horizontal Road
  - 9.2.2 Optimal Braking Performance Under Steering
    - 9.2.2.1 Front Lock-Up
    - 9.2.2.2 Rear Lock-Up
- 9.3 Steering and Acceleration
  - 9.3.1 Simple Acceleration and Steering on a Horizontal Road
  - 9.3.2 Optimal Acceleration Performance Under Steering
    - 9.3.2.1 Front Skid
    - 9.3.2.2 Rear Skid
- 9.4 Vehicle Critical Speed
- 9.5 Vehicle Stability
- 9.6 Summary
- 9.7 References

## Chapter 10 Accident Reconstruction

- 10.1 Introduction and Objectives
- 10.2 Basic Equations of Motion
- 10.3 Drag Factor and Coefficient of Friction
- 10.4 Work, Energy, and the Law of Conservation of Energy
- 10.5 Driver Perception and Response
- 10.6 Engineering Models and Animations
  - 10.6.1 Function of Accident Scene Models
  - 10.6.2 Model Application
  - 10.6.3 Reconstruction Animations
- 10.7 Lane Change Maneuver Model
- 10.8 Speed Estimates for Fall, Flip, or Vault

- 10.8.1 Fall
- 10.8.2 Flip
- 10.8.3 Vault
- 10.9 Speed Estimates from Yaw Marks
- 10.10 Impact Analysis
  - 10.10.1 Straight Central Impact
  - 10.10.2 Noncentral Collisions
  - 10.10.3 Crush Energy and  $\Delta V$
- 10.11 Vehicle-Pedestrian Collisions
  - 10.11.1 Pedestrian Trajectories
  - 10.11.2 Mathematical and Hybrid Models
- 10.12 Accident Reconstruction Software
  - 10.12.1 Software Acronyms: REC-TEC with DRIVE<sup>3</sup> and MSMAC<sup>RT</sup>
  - 10.12.2 VCRware
  - 10.12.3 CRASHEX
  - 10.12.4 ARSoftware
  - 10.12.5 Engineering Dynamics Corporation
  - 10.12.6 Macinnis Engineering Associates (MEA) and MEA Forensic Engineers & Scientists
  - 10.12.7 Maine Computer Group
  - 10.12.8 McHenry Software, Inc
  - 10.12.9 Software Acronym: VDANL
  - 10.12.10 Expert AutoStats® ~ Vehicle Dimension- Weight-Performance Data
  - 10.12.11 Other Accident Reconstruction Software Sites
- 10.13 Low-Speed Sideswipe Collisions
  - 10.13.1 Funk-Cormier-Bain Model
  - 10.13.2 Modeling Procedure
- 10.14 Summary
- 10.15 References

## Appendix A Vector Algebra

- A.1 Real and Complex Vectors
- A.2 Laws of Vector Operation
- A.3 Linear Dependence
- A.4 Three-Dimensional Vectors
- A.5 Properties of the Scalar Product of Vectors
- A.6 Direction Angles
- A.7 Vector Product
- A.8 Derivative of a Vector
- A.9 References

## Appendix B Matrix Analysis

- B.1 Introduction
- B.2 Definitions of Matrices
- B.3 Matrix Operations

# Road Vehicle Dynamics

- B.4 Matrix Inversion
- B.5 Determinants
- B.6 More on Matrix Inversion
- B.7 System of Algebraic Equations
- B.8 Eigenvalues and Eigenvectors
- B.9 Quadratic Forms
- B.10 Positive Definite Matrix
- B.11 Negative Definite Matrix
- B.12 Indefinite Matrix
- B.13 Norm of a Vector
- B.14 Partitioning of Matrices
- B.15 Augmented Matrix
- B.16 Matrix Calculus
- B.17 Summary
- B.18 References
- B.19 Glossary of Terms

## Appendix C Laplace Transforms

- C.1 Laplace Transformation
- C.2 Existence of Laplace Transform
- C.3 Inverse Laplace Transform
- C.4 Properties of the Laplace Transform
  - C.4.1 Multiplication by a Constant
  - C.4.2 Sum and Difference
- C.5 Special Functions
  - C.5.1 Exponential Function
  - C.5.2 Step Function
  - C.5.3 Ramp Function
  - C.5.4 Pulse Function
  - C.5.5 Impulse Function
  - C.5.6 Dirac Delta Function
  - C.5.7 Sinusoidal Function
- C.6 Multiplication of  $f(t)$  by  $e^{-at}$
- C.7 Differentiation
- C.8 Integration
- C.9 Final Value Theorem
- C.10 Initial Value Theorem
- C.11 Shift in Time
- C.12 Complex Shifting
- C.13 Real Convolution (Complex Multiplication)
- C.14 Inverse Laplace Transformation
  - C.14.1 Partial Fraction Expansions
  - C.14.2 Case I-Partial Fraction Expansion When  $Q(s)$  Has Distinct Roots
  - C.14.3 Case II-Partial Fraction Expansion When  $Q(s)$  Has Complex Conjugate Roots
  - C.14.4 Case III-Partial Fraction Expansion When  $Q(s)$  Has Repeated Roots
- C.15 Solution of Differential Equations

- C.16 Summary
- C.17 References

## Appendix D Glossary of Terms

## Appendix E Direct Numerical Integration Methods

- E.1 Introduction
- E.2 Single-Degree-of-Freedom System
  - E.2.1 Finite Difference Method
  - E.2.2 Central Difference Method
  - E.2.3 Runge-Kutta Method
- E.3 Multiple-Degrees-of-Freedom System
- E.4 Explicit Schemes
  - E.4.1 Central Difference Method
  - E.4.2 Fourth-Order Runge-Kutta Method
- E.5 Implicit Schemes
  - E.5.1 Houbolt Method
  - E.5.2 Wilson- $\theta$  Method
  - E.5.3 Newmark- $\beta$  Method
- E.6 Case Studies
  - E.6.1 Linear Dynamic System
  - E.6.2 Nonlinear Dynamic System
- E.7 Summary
- E.8 References

## Appendix F Units and Conversion

- F.1 The S.I. System of Units
- F.2 S.I. Unit Prefixes
- F.3 S.I. Conversion
- F.4 References

## Appendix G Accident Reconstruction Formulae

- G.1 Center of Mass
- G.2 Slide-to-a-Stop Speed
- G.3 Yaw, Sideslip, and Critical Curve Speed
- G.4 Combined Speeds
- G.5 360-Degree Momentum Speed Analysis
- G.6 Tip and Rollover Speed
- G.7 Weight Shift and Speed
- G.8 Kinetic Energy and Speed
- G.9 Fall, Slip, and Vault Speeds

## Bibliography

## List of Symbols

## Index

## About the Authors